BUBBLE DYNAMICS AND INTERFACE PHENOMENA
Aims and Scope of the Series

The purpose of this series is to focus on subjects in which fluid mechanics plays a fundamental role.

As well as the more traditional applications of aeronautics, hydraulics, heat and mass transfer etc., books will be published dealing with topics which are currently in a state of rapid development, such as turbulence, suspensions and multiphase fluids, super and hypersonic flows and numerical modelling techniques.

It is a widely held view that it is the interdisciplinary subjects that will receive intense scientific attention, bringing them to the forefront of technological advancement. Fluids have the ability to transport matter and its properties as well as transmit force, therefore fluid mechanics is a subject that is particularly open to cross fertilisation with other sciences and disciplines of engineering. The subject of fluid mechanics will be highly relevant in domains such as chemical, metallurgical, biological and ecological engineering. This series is particularly open to such new multidisciplinary domains.

The median level of presentation is the first year graduate student. Some texts are monographs defining the current state of a field; others are accessible to final year undergraduates; but essentially the emphasis is on readability and clarity.

For a list of related mechanics titles, see final pages.
Bubble Dynamics and Interface Phenomena

Proceedings of an IUTAM Symposium held in Birmingham, U.K.,
6–9 September 1993

Edited by
J. R. BLAKE
J. M. BOULTON-STONE

and

N. H. THOMAS

School of Mathematics and Statistics,
University of Birmingham,
Edgbaston, Birmingham, U.K.

SPRINGER SCIENCE+BUSINESS MEDIA, B.V.
Contents

Preface ix
List of participants xi

Part One: Bubble dynamics and bubble interactions

Bubble dynamics: Some things we did not know ten years ago. Prosperetti, A. 3
Experimental studies of bubble collapse. Field, J.E. 17
Dynamics of cavitation bubble interactions. Robinson, P.B. & Blake, J.R. 55
Vortex simulations of bubble oscillations. Oguz, H.N. 65
Self-propulsion of bubbles in a weakly nonuniform flow field. Galper, A. & Miloh, T. 73
The dynamics of cavity clusters in polymer aqueous solutions subjected to an oscillating pressure. Shima, A. & Tsujino, T. 81
Dynamics of bubbly clusters and free surfaces at shock wave reflection. Besov, A & Kedrinskii, V. 93

Part Two: Sound and wave propagation

On equations of dynamics of bubbly liquids. Nigmatulin, R.I. 121
Equations describing the propagation of nonlinear modulation waves in bubbly liquids. Gumerov, N.A. 131
Linear wave propagation in bubbly liquids with a continuous bubble size distribution. Graviluyk, S.L. 141
Active and passive acoustic roles of bubbles in the ocean. Yoon, S.W. & Choi, B.K. 151
Transient bubble oscillations associated with the underwater noise of rain detected optically and some properties of light scattered by bubbles. Stroud, J.S. & Marston, P.L. 161
Cavitation nuclei and thresholds of acoustic cavitation in ocean water. Akulichev, V.A. 171
## Part Three: Bubbles in flow

Bubble dynamics and the sound emitted by cavitation. van Wijngaarden, L. 181

Strong bubble/bubble and bubble/flow interactions. Chahine, G.L. 195

Comparison of observed and calculated shapes of travelling cavitation bubbles. Kuhn de Chizelle, Y. & Brennen, C.E. 207

**Patch cavitation in flow past a rigid body.** Howison, S., Morgan, J. & Ockendon, J. 219


Observations of cloud cavitation on a stationary 2D profile. de Lange, D.F., de Bruin, G.J. & van Wijngaarden, L. 241

### Numerical simulations of rising bubbles

Esmæeli, A., Ervin, E. & Tryggvason, G. 247


Role of coherent structures in bubble transport by turbulent shear flows. Sene, K.J., Hunt, J.C.R., Perkins, R.J. & Thomas, N.H. 269

A new design of the cavitation susceptibility meter: The venturix. Pham, T.M., Michel, J.M. & Lecoffre, Y. 277

## Part Four: Sonoluminescence, acoustic cavitation and ultrasound


Approaching bubble dynamics with lasers, holography and computers. Lauterborn, W., Eick, I. & Philipp, . 299

Pressure measurements during acoustic cavitation by sonoluminescence. Kemper, K.A. 311

Bubbles deformation and interface disruption as a source of sonochemical and sonoluminescent activity. Lepoint, T., Voglet, N., Faille, L. & Mullie, F. 321

Sonochemistry — The chemical uses of cavitation. Mason, T.J. 335


Surface mode deformations on an oscillating bubble. Shaw, S.J. 355

Visualisation of laser-induced vapor bubbles and pressure waves. Alloncle, A.P., Dufresne, D. & Autric, M. 365

## Part Five: Jet impact and underwater explosions

Low-pressure zones under a liquid-solid impact. Korobkin, A.A. 375

Inertial shocks in surface waves and collapsing bubbles Longuet-Higgins, M.S. 383
**Contents**

Jet formation at a free surface. *Peregrine, D.H. & Prentice, P.R.* 397

The rebound of toroidal bubbles. *Best, J.P.* 405

Energy losses in non-classical free surface flows. *Szymczak, W.G.* 413


The behavior of a cavitation bubble near a rigid wall. *Zhang, S., Duncan, J.H. & Chahine, G.L.* 429

Numerical simulation of the interaction of an explosion bubble with a submerged spherical pressure vessel. *Duncan, J.H. & Milligan, C.D.* 437

**Part Six: Bursting bubbles, coalescence and internal phenomena**

Bouncing and coalescence phenomena of two bubbles in water. *Duineveld, P.C.* 447

**Bursting of bubbles stabilised by surfactans for control of cell damage.** *Kowalski, A.J. & Thomas, N.H.* 457

Internal phenomena in bubble motion. *Takekura, F. & Matsumoto, Y.* 467

Investigations into the use of two frequency excitation to accurately determine bubble sizes. *Phelps, A.D. & Leighton, T.G.* 475

Index of authors and papers 485
Preface

This volume contains papers presented at the IUTAM Symposium on Bubble Dynamics and Interface Phenomena held at the University of Birmingham from 6–9 September 1993. In many respects it follows on a decade later from the very successful IUTAM Symposium held at CALTECH in June 1981 on the Mechanics and physics of bubbles in liquids which was organised by the late Milton Plesset and Leen van Wijngaarden. The intervening period has seen major development with both experiment and theory. On the experimental side there have been advances with very high speed photography and data recording that provide detailed information on fluid and interface motion. Major developments in both computer hardware and software have also led to extensive improvement in our understanding of bubble and interface dynamics although development is still limited by the sheer complexity of the laminar and turbulent flow regimes often associated with bubbly flows.

The symposium attracts wide and extensive interest from engineers, physical, chemical, biological and medical scientists and applied mathematicians. The scientific committee sought to achieve a balance between theory and experiment over a range of fields in bubble dynamics and interface phenomena. It was our intention to emphasise both the breadth and recent developments in these various fields and to encourage cross-fertilisation of ideas on both experimental techniques and theoretical developments. The programme, and the proceedings recorded herein, cover bubble dynamics, sound and wave propagation, bubbles in flow, sonoluminescence, acoustic cavitation, underwater explosions, bursting bubbles and ESWL.

The Symposium was officially opened by Professor Julian Hunt, Chief Executive of the Meteorological Office. Birmingham is only a short distance from Stratford-upon-Avon, Shakespeare’s home, so it was fitting that appropriate quotes should emerge; examples being

*The Earth hath bubbles, as the water has*

and

*Double, double toil and trouble, fire burn and cauldron bubble.*

The first quotation was attributed to the Scottish industrialist Baron Macbeth and his interest in the off-shore oil industry and the need to study two phase flow in oil wells while the second apparent connection was in association with the importance of bubbles in chemical engineering and biotechnology and the need to apply scaling arguments! Clearly with an opening address beginning with such style, the Symposium was off to a brilliant start and that proved to be the case. The theme of *est homo* (merely) *bulla* was expanded upon to an even greater extent by Professor Andrea Prosperetti during a lavishly illustrated after-dinner speech.
The plenary session at the end of the conference proved a period for lively and animated debate. Although a number of topics were raised, two topics led to intense debate. They were (i) the physics and chemistry of sonoluminescence and (ii) the modelling of jet impact during axisymmetric bubble collapse. The proceedings have been arranged so that the relevant papers on these two topics follow each other. The reader of this volume can thus see the differing views on sonoluminescence presented in these papers. The discussion on jet impact centred around representation of the vortex sheet, ranging from papers which omitted a vortex sheet entirely thus conserving momentum and energy, through to algorithms which do include a vortex sheet losing momentum and energy from the liquid. Both these topics need further development and scientific understanding and no doubt will be reported in the scientific press over the next year or so.

Finally it is my pleasant duty as Chair of the Scientific Committee to express my grateful thanks to: Mrs. June Brough who was Secretary for the Symposium, the Scientific Committee, the Local Organising Committee and my two associate editors of the Proceedings, Dr. Jeremy Boulton-Stone and Dr. Neale Thomas. Finally my special thanks to IUTAM for awarding the Symposium to Birmingham. May the benefits of this Symposium be found in future scientific literature and ultimately lead to the betterment of society and mankind.

John Blake,
March 1994.
List of participants

*IUTAM SYMPOSIUM on Bubble Dynamics and Interface Phenomena, held in Birmingham, UK, 6–9 September, 1993*

Dr V A Akulichev
Inst. of Marine Tech Problems
Russian Academy of Sciences
5a Sukhanov Street
Vladivostok 690600
Russia

Dr A P Alloncle
Institut de Mécanique des Fluides
Parc Scientifique et Technologique
163 Avenue de Luminy
13009 Marseille
France

Dr F Avellan
IMHF–EPFL
33 Avenue de Cour
CH 1007 Lausanne
Switzerland

Ms I Baquet
IEMN–LOAE
Université de Valenciennes
B.P. 311
59034 Valenciennes Cedex
France

Dr J P Best
Underwater Systems Division
Materials Research Laboratory
P.O. Box 50
Ascot Vale VIC 3032
Australia

Dr A Biesheuvel
Dept. of Mech. Eng.
Twente University
Postbus 217
7500 AE Enschede
The Netherlands

Professor J R Blake
School of Mathematics & Statistics
The University of Birmingham
Edgbaston
Birmingham B15 2TT
U.K.

Dr J M Boulton-Stone
School of Mathematics & Statistics
The University of Birmingham
Edgbaston
Birmingham B15 2TT
U.K.

Dr N K Bourne
PCS, Cavendish Laboratory
University of Cambridge
Madingley Road
Cambridge CB3 0HE
U.K.

Dr C E Brennen
Mechanical Engineering
104–44 California Inst. of Tech.
Pasadena, CA 91125
U.S.A.

Ir G J de Bruin
Twente University, WB/W–250
Postbus 217
7500 AE Enschede
The Netherlands
Dr G L Chahine  
Dynaflow Inc  
7210 Pindell School Road  
Fulton, MD 20901  
U.S.A.

Professor A K Chesters  
La Couscouillette  
Montlaur  
11220 Lagrasse (Aude)  
France

Dr M Cooker  
School of Mathematics  
University of East Anglia  
Norwich  
Norfolk  
U.K.

Dr A J Croft  
School of Math. & Info. Sciences  
Coventry University  
Priory Street  
Coventry CV1 5FB  
U.K.

Professor L A Crum  
Applied Physics Laboratory  
University of Washington  
1013 N.E. 40th Street  
Seattle, WA 98105  
U.S.A.

Mr P C Duineveld  
Twente University  
Facult. WB  
Postbus 217  
7500 AE Enschede  
The Netherlands

Dr J H Duncan  
Department of Mech. Eng.  
University of Maryland  
College Park, MD 20742  
U.S.A.

Dr D C Emmony  
Department of Physics  
Loughborough University  
Loughborough  
Leicester LE11 3TU  
U.K.

Dr J Field  
PCS, Cavendish Laboratory  
University of Cambridge  
Madingley Road  
Cambridge CB3 0HE  
U.K.

Mr A Galper  
Department of Mech. Eng.  
Tel Aviv University  
Tel Aviv  
Israel

Dr S L Gavriliyuk  
Laurentyev Inst. of Hydrodynamics  
Novosibirsk 630090  
Russia

Dr P Giovine  
Mathematics Inst.  
Eng. Faculty, University  
Via E.Cuzzocrea, 48  
89128 Reggio Calabria  
Italy

Dr N A Gumerov  
177 Winthrop Road #3  
Brookline, MA 02146  
U.S.A.

Dr J Gunson  
School of Mathematics & Statistics  
The University of Birmingham  
Edgbaston  
Birmingham B15 2TT  
U.K.
Mr A Hardwick
P & C
Cavendish Laboratory
Madingley Road
Cambridge
U.K.

Dr G Harris
US Naval Surface Warfare Center
Dahlgren Division
White Oak Detachment
Silver Spring, MD 20903
U.S.A.

Dr P Harris
Dept. of Mathematical Sciences
University of Brighton
Lewes Road
Moulscomb
Brighton BN2 4GJ
U.K.

Mr M Hooton
School of Mathematics & Statistics
The University of Birmingham
Edgbaston
Birmingham B15 2TT
U.K.

Professor J C R Hunt
Meteorological Office
London Road
Bracknell
Berkshire. RG12 2SZ
U.K.

Mrs Y H Jin
Department of Physics
Loughborough University
Loughborough
Leics. LE11 3TU
U.K.

Mr P Jones
Dantec Electronics Ltd
Techno House
Redcliffe Way
Bristol BS1 6NU
U.K.

Ms K Jungnickel
Medical Laser Center Lubeck
Peter-Monnik-Weg 4
W-2400 Lubeck
Germany

Mr R Kay
Fluid Gravity Eng.
Chiltlee Manor
Haslemere Road
Liphook
Hants GU30 7AZ
U.K.

Professor V Kedrinskii
Lavrentyev Inst of Hydrodynamics
Russian Academy of Sciences
Siberian Division
Novosibirsk 630090
Russia

Mr G S Keen
School of Mathematics & Statistics
The University of Birmingham
Edgbaston
Birmingham B15 2TT
U.K.

Dr N S Khabeev
Inst. of Mechanics
Moscow University
Michurinski pr 1
Moscow 117192
Russia
Dr T Kodama  
The Inst. of Fluid Science  
Tohoku University  
Katahira, Aoba-ku  
Sendai 980  
Japan

Dr A A Korobkin  
Dept. of Marine Hydrodynamics  
Norwegian Institute of Technology  
N–7034 Trondheim–NTH  
Norway

Dr A J Kowalski  
Unilever Research  
Port Sunlight Laboratory  
Bebington  
Wirral L63 3JW  
U.K.

Mr D F de Lange  
Twente University  
Postbus 217  
7500 AE Enschede  
The Netherlands

Ir J H Lammers  
University of Twente, WB/WS  
Postbus 217  
7500 AE Enschede  
The Netherlands

Professor W Lauterborn  
Institut für Angewandte Physik  
TH Darmstadt  
Schloßgartenstr 7  
D–64289 Darmstadt  
Germany

Dr T G Leighton  
Inst of Sound & Vibration Research  
University of Southampton  
Highfield  
Southampton SO9 5NH  
U.K.

Professor T LePoint  
CERIA, Institut Meurice  
Avenue Emile Gryzon 1  
B–1070 Bruxelles  
Belgium

Professor M Longuet-Higgins  
Inst. for Nonlinear Science  
University of California, San Diego  
La Jolla, CA 92093–0402  
U.S.A.

Dr K Maeno  
Institut de Mécanique des Fluides  
Univ d’Aix Marseille II  
163 Avenue de Luminy  
13009 Marseille  
France

Dr P L Marston  
Department of Physics  
Washington State University  
Pullman, WA 99164  
U.S.A.

Professor T J Mason  
School of Natural & Environmental Science  
Coventry University  
Coventry CV1 5FB  
U.K.

Dr F M J McCluskey  
Laboratoire d’Electrostatique  
C.N.R.S.  
25 Avenue des Martyrs  
B.P. 166  
38042 Grenoble Cedex 9  
France

Dr A M Milne  
Fluid Gravity Eng.  
Chilteem Manor  
Haslemere Road  
Liphook  
Hants GU30 7AZ  
U.K.
List of participants

Dr J Morgan
OCIAM
Mathematical Inst.
24–29 St Giles’
Oxford OX1 3LB
U.K.

Dr F Mullie
CERIA, Institut Meurice
Avenue Emile Gryzon 1
B–1070
Belgium

Dr M Nagata
School of Mathematics & Statistics
The University of Birmingham
Edgbaston
Birmingham B15 2TT
U.K.

Professor R I Nigmatulin
Dept. of Nuclear Eng. & Eng. Physics
Reusselaer Polytechnic Institute
Troy, NY 12180–3590
U.S.A.

Dr T Obara
PCS, Cavendish Laboratory
University of Cambridge
Madingley Road
Cambridge CB3 0HE
U.K.

Mr I Ogilvy
Defence Research Agency
Admiralty Research Establishment
St Leonard’s Hill
Dunfermline
Fife, KY11 5PW
U.K.

Dr H N Oğuz
Department of Mech. Eng.
Johns Hopkins University
Baltimore, MD 21218
U.S.A.

Professor D H Peregrine
School of Mathematics
University Walk
Bristol BS8 1TW
U.K.

Dr R J Perkins
DAMTP
Silver Street
Cambridge CB3 9EQ
U.K.

Mrs T M Pham
LEGI/IMG
B.P. 53 X
38041 Grenoble Cedex
France

Mr A Phelps
Inst of Sound & Vibration Research
University of Southampton
Southampton SO9 5NH
U.K.

Professor A Prosperetti
Department of Mech. Eng.
Johns Hopkins University
Baltimore, MD 21218
U.S.A.

Mr D Ramble
ISVR
University of Southampton
Highfield
Southampton SO9 5NH
U.K.

Mr D V Ritzel
Ship Structures & Materials Divn
Materials Research Laboratory
P.O. Box 50
Ascot Vale, VIC 3032
Australia
List of participants

Dr P Robinson
School of Mathematics & Statistics
The University of Birmingham
Edgbaston
Birmingham B15 2TT
U.K.

Dr R Roy
Applied Physics Laboratory
University of Washington
1013 NE 40th Street
Seattle, WA 98105
U.S.A.

Dr S J Shaw
School of Mathematics & Statistics
The University of Birmingham
Edgbaston
Birmingham B15 2TT
U.K.

Professor A Shima
Inst. of Fluid Science
Tohoku University
2-1-1 Katahira, Aoba-Ku
Sendai 980
Japan

Mire S Sochard
Laboratoire de Genie Chimique
(NRS, URA)
Chemin de la loge
31078 Toulouse Cedex
France

Dr W-K Soh
Department of Mech. Eng.
University of Wollongong
Wollongong, NSW 2500
Australia

Dr P D M Spelt
University of Twente, WB/WS
Postbus 217
7500 AE Enschede
The Netherlands

Professor K S Suslick
School of Chemical Sciences
Noyes Lab
Univ. of Illinois at
Urbana-Champaign
505 S Mathews Avenue
Urbana, IL 61801
U.S.A.

Dr W G Szymczak
Naval Surface Warfare Center
Code R44
10901 New Hampshire Avenue
Silver Spring, MD 20903-5000
U.S.A.

Dr F Takemura
Department of Mech. Eng.
University of Tokyo
7-3-1 Hongo, Bunkyo-ku
Tokyo 113
Japan

Dr N H Thomas
School of Chemical Eng.
The University of Birmingham
Edgbaston
Birmingham B15 2TT
U.K.

Dr Y Tomita
Hakodate College
Hokkaido University of Education
1-2 Hachiman-cho
Hakodate 040
Japan

Dr R P Tong
School of Mathematics & Statistics
The University of Birmingham
Edgbaston
Birmingham B15 2TT
U.K.
Miss M Topliss  
Rm3–1 Mathematics Department  
Bristol University  
Bristol BS8 1TW  
U.K.

Dr G Tryggvason  
2250 GG Brown  
University of Michigan  
Ann Arbor, MI 48109–2125  
U.S.A.

Dr A Vogel  
Medical Laser Centre Lubeck  
Peter-Monnik-Weg 9  
D–2400 Lubeck 1  
Germany

Professor C A Ward  
Department of Mech. Eng.  
University of Toronto  
5 King’s College Road  
Toronto, M5S 1A4  
Canada

Dr P N Ward  
Zeneca Bio Products  
P.O. Box 2  
Belasis Avenue  
Billingham  
Cleveland TS23 1YN  
U.K.

Professor L van Wijngaarden  
University Twente  
Postbus 217  
7500 AE Enschede  
The Netherlands

Professor S W Yoon  
Sung Kyun Kwan University  
300 Chunchun-dong  
Jangan-ku  
Suwon 440–746  
Repub of Korea

Dr B Zequri  
National Physical Laboratory  
Teddington  
U.K.

Dr S Zhang  
Department of Mech. Eng.  
University of Maryland  
College Park, MD 20742  
U.S.A.
Jet impact and underwater explosions
Low-pressure zones under a liquid-solid impact

A.A. Korobkin
Laurentiev Institute of Hydrodynamics, Novosibirsk, 630090, Russia

Abstract. The experiments of J. H. Brunton and J. E. Field (Cavendish Laboratory) have shown that an intense cavitation occurs at a solid surface under its impact with a liquid drop. This is due to the interaction of the relief waves which expand from the free surface of the liquid.

The method making it possible to evaluate and to analyse in detail the pressure field inside the liquid within the framework of the classical acoustic approximation is suggested. It is shown that the possibility of the appearance of low-pressure zones is determined by the geometries of both the rigid surface and the liquid domain. For example, the rarefaction zones are absent under the impact of a parabolic contour on the plane free surface

Both the plane and axisymmetric problems of a jet impact onto a rigid plane are considered. The exact solutions of the problems are presented. For the axisymmetric case the focusing of compression waves inside the jet is shown to lead to an infinite positive values of the pressure. On the other hand, the focusing of the relief waves leads to finite negative values.

The new nonclassical conditions on the impacted surface are suggested. The conditions have the form of the one-side inequalities: i) the pressure on the impacted surface is nonnegative; ii) the liquid particles can move from the wall but cannot penetrate it. The general method of treating the liquid-solid impact problem with such modified condition on the rigid surface is presented.

The modified problem of a plate impact onto the initially plane free surface of a highly compressible liquid is considered. The initial stage of the cavity formation due to the interaction of the relief wave is described in quadratures.

1 Introduction

The well-known experiments on a collision of liquid drops with rigid surfaces were carried out by J.H. Brunton and J.E. Field from the 1950's onwards. Innovative techniques allowed them to get clear photographs of the processes proceeding inside a liquid drop during impact (see, for example, Brunton and Camus 1970, Brunton and Rochester 1979, Field et.al. 1985). One of their results was the observation of an intense cavitation both inside the drop and on the impacted surface. This discovery was interesting because usually one connects a liquid-solid impact only with large hydrodynamics loads. However, it was found that not only compression waves but rarefaction waves are also formed under the impact. The latter are provided by the presence of the liquid free surface and they move from the periphery of the contact region between the rigid surface and the drop to its centre. The pressure drops in the region of the interaction of the rarefaction waves and can become so much lower that the liquid starts to disintegrate with the appearance of cavities and/or to separate from the impacted surface (interface cavitation). Then the pressure increases again, which leads to a collapse of the bubbles with the formation of new compression waves. The collapse of the bubbles adjacent to the surface is of great interest in the context of possible erosion damage by the interface cavitation. The interface cavitation does not happen consistently, its appearance depends on the adhesive properties of the liquid. Brunton (1971)
observed the liquid separation from the impacted surface for water, but for silicone oils bubbles were close to the surface but not adjacent to it. As pointed out by Field (1971), the analysis of the cavitation under a liquid-solid impact is very important for a low-speed impact, when the hydrodynamic pressure is not too large and does not correspond clearly to the observed damage of the rigid surface.

To describe the interface cavitation, let us assume:
1. a liquid be ideal and compressible;
2. an impact velocity be small compared to the velocity of sound in the liquid at rest;
3. a liquid be represented as a continuous medium with no phase changes even at the lowest pressures;
4. any connection forces between the liquid particles and the rigid surface be absent;
5. an impacted surface be rigid, undeformable and impenetrable for the liquid particles.

The present study is focused on the liquid flow and the pressure distribution inside the region of the first rarefaction waves interaction. During the stage under consideration the displacement of the rigid surface and the free surface deformation are small and can be neglected. The nonlinear terms in the motion equations and the boundary conditions therewith are not taken into account. The boundary conditions are taken on the initially undisturbed boundary of the liquid domain. As a result one obtains the acoustic approximation well-known in the liquid-solid impact theory.

The main goal of this paper is to demonstrate that after some modifications the acoustic approximation may describe many observed phenomena, including complicated ones such as the interface cavitation, i.e. the appearance of a new free surface.

2 Blunt-body impact on a free surface

If both the rigid surface and the surface of the liquid just before the impact are smooth then at the initial stage the liquid flow will be approximately the same as in the problem of an appropriate blunt-body impact on the liquid half-plane (see Korobkin, 1992b).

Within the framework of the acoustic approximation the flow domain coincides with the half-plane $y < 0$ which is occupied by the liquid at the initial moment. The liquid flow is described by the velocity potential $\phi(x, y, t)$, for which the initial boundary-value problem in the non-dimensional variables has the form:

$$\phi_{tt} = \phi_{xx} + \phi_{yy} \quad (y < 0),$$
$$\phi = 0 \quad (y = 0, |x| > a(t)), $$
$$\phi_y = -1 \quad (y = 0, |x| < a(t)).$$
The non-dimensional variables are chosen in such a way that the sound velocity in the liquid at rest, the impact velocity and the liquid density are equal to unity. The boundary part $-a(t) < x < a(t), y = 0$ corresponds to the contact region of the entering contour with the liquid, and the boundary parts $x < -a(t)$ and $x > a(t)$ correspond to the free surface where the pressure is zero for all time. The points with the coordinates $x = \pm a(t), y = 0$ correspond to the contact points of the free liquid boundary with the surface of the rigid body. The function $a(t)$ describes the motion law of the points and is assumed to be determined in advance. The method of its calculation has been suggested earlier (see Korobkin, 1992a).

For a blunt body there is an instant $T$ such that for $0 < t < T$ the free surface is undisturbed, and the shock front is attached obliquely to the contact points. After escaping the shock front on the free surface, the problem using the acoustic approximation can be divided into two parts. Firstly, the evolution of the free surface and the positions of the contact points are determined. Then to calculate the pressure field and the liquid flow, the parallels between the impact problem and the problem of a plane lifting surface placed in a supersonic stream at a small angle of attack are used. This approach makes it possible to analyze analytically the characteristics of the process.

In the general case, the position of the contour in the nondimensional variables are given by the equation $y = M(f(x) - t)$, where $M$ is the Mach number, the function $f(x)$ describes the contour form. In the region of the interaction of the rarefaction waves the pressure $p$ is given by the formula

$$p(x, y, t) = v_c(t_1)F_1(x, y, t) + v_c(t_2)F_2(x, y, t) - \frac{1}{\pi} \int_{\xi_L}^{\xi_L'} \frac{d\xi}{\sqrt{(t - f(\xi))^2 - (x - \xi)^2 - y^2}}.$$  

Here $v_c(t)$ is the velocity of the contact point at the instant $t, T < t_1(x, y, t), t_2(x, y, t) < t$, the functions $F_1(x, y, t), F_2(x, y, t)$ are bounded and positive, $|\xi_L|, |\xi_L'| < a(T)$ (see Korobkin, 1994).

It should be noted that for an arbitrary contour we can indicate inside the flow region a curve below which the pressure is positive at all times. The curve is given by the equation $\xi'_L(x, y, t) = \xi_L(x, y, t)$. There is $\xi'_L < \xi_L$ above this curve, therefore the integral is positive and it is possible that the pressure may be negative. The possibility of this event depends on the magnitude of the velocity of the contact points $v_c$. More exactly, this velocity must be less than some critical value. To clarify this statement, let us consider the entry of a rectangular cylinder, with a curved base of radius of curvature $R$. Then one has $v_c(t) = 0$ when $t > T_R > T$, where $T_R$ depends on the radius of the circle segment. In this case two first summands in the formula are zero and, hence, the pressure is negative. In particular, for a parabolic contour it is shown that the pressure in the region concerned is positive.
The result can be formulated as follows: the appearance of the negative pressure zones under the liquid-solid impact depends on the speed of the extension of the contact spot and is impossible when the speed is sufficiently high.

In the cases when the contact spot does not extend and its size is independent of time, as in the problem of a jet impact on a rigid plane or a disk impact on a liquid half-plane, the negative-pressure zones can be indicated for any impact velocity. The exact positions of the zones and the pressure distribution inside them are of great importance in context of a possible occurrence of the interfacial bubbles.

3 Jet impact onto a rigid plane

The plane and the axisymmetric problems of a jet impact onto a rigid surface are considered. At the initial moment the liquid touches the rigid surface and is at rest, then the rigid plane starts to move with a constant velocity. The shock wave and the relief ones are generated simultaneously. The pressure distribution inside the jet is given by the formulae

Plane jet:

\[
p(r, z, t) = \frac{4}{\pi} \left\{ \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} \cos[\pi r(k + 0.5)]J_0[\pi \sqrt{t^2 - z^2}(k + 0.5)] \right\} H(t - z).
\]

Axisymmetric jet:

\[
p(r, z, t) = 2 \left\{ \sum_{k=1}^{\infty} \frac{J_0(r \mu_k)}{\mu_k J_1(\mu_k)} J_0[\sqrt{t^2 - z^2}\mu_k] \right\} H(t - z), \quad J_0(\mu_k) = 0.
\]

These formulae are very convenient for the calculations of some integral characteristics and for the analysis of the pressure peculiarities too. The plane problem has been analyzed by Frankel (1990) using a different method. General theory of a jet impact onto a rigid plane for an arbitrary geometry of the jet cross-section and the application of the theory to the problem of energy loss under the impact will be published in the near future.

In the axisymmetrical case both compression and rarefaction waves are torus-like, therefore under their interaction an extremely low or high values of the pressure respectively can be expected. To clarify this point, it will suffice to consider the pressure at the centre of the contact spot only. Separating the main part of the solution one can write

\[
p(0, 0, t) = \frac{2}{\pi \sqrt{t}} \ln[\cos(\pi (t - 2N)/2) \cos(\pi (t + 1)/4) + p_r(0, 0, t),
\]

where the regular component \( p_r(0, 0, t) \) is a bounded function, the integer number \( N \) is such that \( |t - 2N| < 1 \). This form of the solution indicates clearly that while interacting the relief waves the pressure can be very low but finite, whereas
the interaction of the compression waves leads to the logarithmic growth of the pressure.

We can conclude that even although the low-pressure zones arise under a liquid-solid impact, the pressure inside them is bounded. Hence, the acoustic approximation might be expected to describe the real liquid flow with regards to the interface phenomena.

4 Bubble formation on the impacted rigid surface

Within the framework of the above problem for the velocity potential, the liquid particles on the contact region are adjacent to the rigid surface of the impacting body for all times however large the stretching stresses generated under the interaction of the relief waves might be. Physically, this means that the liquid particles, caught on the rigid surface at some instant of time, can move only along it, but cannot leave it. This is referred to as the slip condition. The condition reflects the assumption that the connection forces between the liquid particles and the rigid surface exceed the hydrodynamic stretching stresses for all times. Generally speaking, this cannot be right and the connection forces must be considered in the problem formulation. Let us denote the limiting negative pressure under which the liquid particles remain yet on the rigid surface by $p_{cav}$. Let the position of the liquid boundary be described by the equation $y = M_1(x, t)$. It is suggested to substitute the slip condition by the following one-side inequalities

$$\eta(x, t) \leq f(x) - t, \quad p \geq p_{cav},$$

$$(p - p_{cav})(\eta(x, t) + t - f(x)) = 0.$$

The first inequality implies that the liquid particles cannot penetrate the rigid surface, the second implies that the pressure in the contact region cannot be less than the limiting value. The last equality implies that the contact region is divided into parts of two types: where $p = p_{cav}$ we have $\eta(x, t) \leq f(x) - t$, this means that in these parts the liquid particles can move from the rigid surface with the formation a new free surface; where $p \geq p_{cav}$ we have $\eta(x, t) = f(x) - t$, i.e. the particles are adjacent to the rigid surface here.

Solving the impact problem with the boundary conditions in the forms of the one-side inequalities is very complicated even within the framework of the acoustic approximation. However, we can use additional information on the impact process. For example, we can reason in advance that up to the instant of the relief wave interactions the pressure on the contact spot is positive and cavities are not formed. At this stage, the acoustic approximation can be used in its classical form. The approximation is valid up to the moment when at some point or on some part of the contact region the pressure becomes equal to $p_{cav}$. Then a cavity attached to the rigid surface comes to form. The shape of the cavity should be determined together with the liquid flow and the pressure inside the liquid domain. Inside
the region of the relief waves interaction the pressure distribution is described by a smooth function, that is why one can prove that initially the cavity expands along the contact spot with a supersonic speed. This means that at this stage the longitudinal size of the cavity is known in advance and only its thickness should be determined.

To demonstrate the method, let us consider the problem of a box-like structure impacting onto the half-plane occupied by a slightly compressible liquid. This problem was studied within the framework of the classical acoustic approximation by Ogilvie (1963). For simplicity of the calculations assume $p_{cav} = 0$. In this case, any connection forces between the liquid particles and the rigid surface are absent.

At the moment of the escape of the relief wave onto the opposite free surface part, the pressure is known to be zero over the whole contact spot and becomes negative later on. Hence, at this moment the liquid particles move from the rigid surface over the contact spot. To find the position of this new free surface, we will use the following integral relation

$$p(x, 0, t) = \frac{1}{\pi} \int_{\sigma} \int \frac{p_0(\xi, 0, \tau) d\xi d\tau}{\sqrt{(t - \tau)^2 - (x - \xi)^2}},$$

which establishes the connection between the pressure on the boundary of the lower half-plane and its normal derivative. The momentum equation gives

$$\frac{\partial p}{\partial y} = -\frac{\partial v}{\partial t}(x, 0, t) =: w(x, t).$$

Therefore, in fact, the mentioned integral relation sets up the connection between the pressure and the normal component of the liquid acceleration on the boundary of the liquid domain.

We shall restrict our consideration to the motion of the new free surface in the region $2 < t < 3, |x| < 3 - t$, where having regard to the one-side inequalities we put $p = 0$ and consider the function $w(x, t)$ as the unknown one. Then the mentioned relation is the integral equation which can be written in the form

$$\frac{1}{\pi} \int_{\sigma_1} \int \frac{w(\xi, 0, \tau) d\xi d\tau}{\sqrt{(t - \tau)^2 - (x - \xi)^2}} = -p_c(x, t),$$

Here, $\sigma_1 = \sigma \cap (t > 2), p_c(x, t)$ is the pressure in $\sigma_1$ calculated within the framework of the classical acoustic approximation. The solution of this integral equation is

$$w(x, t) = \frac{4}{\pi} \int_{0}^{\pi} \frac{\sqrt{1 - ((t - 2) \cos \theta - x)^2}}{t^2 - ((t - 2) \cos \theta - 2x)^2} d\theta.$$

In particular, at the instant of starting the cavity rise we get

$$w(x, 2) = \frac{1}{\pi \sqrt{1 - x^2}}.$$
The cavity form is determined by the double integration of the acceleration $w(x,t)$ in time. It is clear that the linear theory is invalid near the contact points, that is why the present approach cannot be applied for analyzing the cavity form in these small zones. The process of the cavity rise is a long-term one, therefore to study the cavity collapse, the numerical realization of the suggested method should be used. The cavity collapse, i.e. the motion of its boundary to the rigid surface, can be assumed to start not from its periphery but at its centre. It follows from the analysis of the right-hand side of the integral equation.

5 Conclusion

It has been shown that:

1. The possibility of the appearance of the low-pressure zones on the contact spot depends on the speed of this spot expansion.

2. The interaction of the rarefaction waves does not lead to an indefinitely low pressure.

3. The simple modification of the acoustic approximation makes it possible to predict the form and the exact position of a cavity attached to the impacted rigid surface while taking the interface phenomena into account.

References


