GENERALIZED ANISOTROPIC NAVIER-STOKES EQUATIONS

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Annotation. In this talk, we consider the evolution problem for the Navier–Stokes equations of non-Newtonian type and with an anisotropic diffusion. It means that the stress tensor is a nonlinear anisotropic tensor function of the strain rate tensor. This set of equations is potentially useful to model fluids that behave as dilatant fluids in several directions and as pseudoplastic in another distinct ones. The existence of weak solutions is proved for the standard initial and boundary value problem. Also the properties of the solutions such as the extinction in a finite time, the exponential time decay and the power time decay are proved. With this respect, we consider the important case of a forces field with different behavior in distinct directions.

INTRODUCTION

Differential equations with anisotropic diffusion have been studied intensively as witness the large number of works published in recent years (see, e.g. [1] and the references cited therein). In the literature of classical Navier-Stokes equations, anisotropic diffusion is used in the context of geophysical flows with the meaning that the vertical viscosity is distinct from the horizontal one (see e.g. [2, 3]). The resolution of the Navier–Stokes with small initial data in suitable anisotropic spaces is also considered in some literature (see [4] and some of the several works where it is cited). There are also some works in which are studied some anisotropic regularity criteria for the Navier–Stokes equations that extend the well-known Serrin’s condition (see [5]). There is still, in the literature, other types of Navier-Stokes equations where anisotropy is considered in other terms (see [2, 3, 6]). The steady version of this problem was analyzed by the authors in the work [7]. The isotropic version of classical Navier–Stokes equations has been studied in the context of Mathematical Fluid Mechanics during the last 50 years. In the work dating back to 1967, Ladyzhenskaya [8] started the study of the existence and uniqueness of weak solutions to an isotropic version of this problem for N=3. A little bit later, Lions [9] improved and extended, to a general dimension N>2, the existence and uniqueness results of [8]. In recent years, these results were extended to almost the range of the power-law index that characterizes pseudoplastic fluids [10–12].

MAIN RESULTS

In the present talk, we start by introducing the main concepts of the anisotropic function spaces and we define the notions of weak solution to our problem we are interested in this work. Next, we prove the existence of weak solutions to our problem and we established an energy inequality which will be fundamental for the investigation of the properties of the solutions we will study in the sequel. Then we prove the property of extinction (vanishing), of the weak solutions, in a finite time if in some (or several, but not necessarily all) directions the fluid acts as pseudoplastic. The mechanical sense of this property is that, if the flow of such a non-Newtonian fluid is generated by the initial data, then in a finite time the fluid becomes immobile. The large time behavior (exponential and power time decay) of the weak solutions is analyzed also, depending these properties on a suitable combination of pseudoplastic directions with dilatant ones. With this respect, we consider the important case of a forces field with possible different behavior in distinct directions. By exploiting several examples, not only in the case of N=2 and N=3, but also for N=4, we show that the results established here improve its isotropic versions in almost all directions or for particular choices of all the diffusion coefficients. Finally, we study some perturbations of the asymptotically stable equilibrium. Although we can find some features of our problem on the applications, our motivation is purely mathematical and it comes from the research program that we are carrying on. In this program we hopefully can use the Navier–Stokes equations with anisotropic diffusion as a regularization process of the isotropic equations in order to improve some results of the latter. In particular, we shall see that, by considering an anisotropic diffusion, we can considerably decrease, in several directions, the lower bound for the existence result. We will see also that for the phenomenons of fast or slow diffusion, related with the properties of extinction in finite time or (infinity) decay in time, respectively, it will be important, in most situations, the behavior in only two distinct directions.
References